hep-th/yymmnnn

Non Supersymmetric Metastable Vacua in $\mathcal{N} = 2$ SYM Softly Broken to $\mathcal{N} = 1$

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We find non-supersymmetric metastable vacua in four dimensional $\mathcal{N} = 2$ gauge theories softly broken to $\mathcal{N} = 1$ by a superpotential term. First we study the simplest case, namely the $SU(2)$ gauge theory without flavors. We study the spectrum and lifetime of the metastable vacuum and possible embeddings of the model in UV complete theories. Then we consider larger gauge group theories with flavors. We show that when we softly break them to $\mathcal{N} = 1$, the potential induced on specific submanifolds of their moduli space is identical to the potential in lower rank gauge theories. Then we show that the potential increases when we move away from this submanifold, allowing us to construct metastable vacua on them in the theories that can be reduced to the $SU(2)$ case.

1 Introduction

Dynamical supersymmetry breaking in a metastable vacuum is an attractive possibility for supersymmetry breaking. Unlike old-fashioned spontaneous supersymmetry breaking one can consider candidate theories with supersymmetric vacua elsewhere in field space. In a beautiful paper [1] showed that this scenario is realized even in simple $\mathcal{N} = 1$ gauge theories like SQCD with massive flavors. Since [1] there has been a lot of activity in the direction of extending the results in field theory [2, 3, 4, 5, 6, 7, 8, 9] and string theory [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] realizations.

It was already pointed out in [1] that it might be interesting to study the system of $\mathcal{N} = 2$ supersymmetric theories softly broken to $\mathcal{N} = 1$ by superpotential terms. $\mathcal{N} = 2$ theories have moduli spaces of vacua. Unlike $\mathcal{N} = 1$, in $\mathcal{N} = 2$ it is possible to compute the Kahler metric on the moduli space exactly. If we add a small superpotential, we can hope that we can still use the exact Kahler metric. This allows us to compute the scalar potential on the moduli space exactly and look for local minima that correspond to metastable non-supersymmetric vacua.

In this paper we study the simplest example, namely pure $\mathcal{N} = 2$ $SU(2)$ gauge theory, softly broken to $\mathcal{N} = 1$ by a superpotential for the scalar field. For appropriate selection of the superpotential a metastable vacuum appears at the origin of the moduli space. We discuss the spectrum of the theory in this vacuum, its lifetime and possible embeddings of our model in a UV complete theory. Then we consider $\mathcal{N} = 2$ theories with gauge groups of higher ranks and with flavors. We show that on specific submanfolds of their moduli space the potential is identical with the potential of lower rank theories. We also show that these submanifolds can be locally stable allowing us to construct metastable vacua on them as in $SU(2)$.

While this paper was being prepared for publication another paper appeared [20], which has overlap with this work.

2 Pure $\mathcal{N}=2$ $SU(2)$ gauge theory

2.1 The metric on the moduli space

The field content of pure $\mathcal{N}=2$ $SU(2)$ gauge theory consists of the gauge field A_μ, a complex scalar ϕ and fermions, all in the adjoint representation of the gauge group. The theory has a moduli space of vacua, in which the gauge group is broken to $U(1)$, that we will refer to as the Coulomb branch. The classical potential for the scalar field ϕ in $\mathcal{N}=2$ SYM without flavors is:

$$V(\phi)=\frac{1}{g^2}Tr\left(\left[\phi,\phi^\dagger\right]\right) \tag{2.1}$$

Setting the potential to zero gives the semi-classical moduli space of vacua, characterized by a complex number multiplying the element of the Cartan subgroup of the gauge group:

$$\phi=\frac{1}{2}\left(\begin{array}{cc} a & 0 \\ 0 & -a \end{array}\right) \tag{2.2}$$

where a is a complex number. However a is not a gauge invariant quantity, so we identify the vacua by the complex number

$$u=Tr\phi^2=\frac{1}{2}a^2 \tag{2.3}$$

Using the powerful constraints of $\mathcal{N}=2$ supersymmetry, one can go beyond the semiclassical analysis and study the full quantum theory. In the seminal paper [21] Seiberg and Witten managed to determine exactly the low energy effective theory on the Coulomb branch. The quantum moduli space turns out to be the complex u-plane with singularities. Classically one expects a singularity at $u=0$ where the $SU(2)$ gauge symmetry is restored. It turns out that quantum mechanically the point $u=0$ is smooth and there is no gauge symmetry enhancement anywhere on the moduli space. Instead, there are two singularities at $u=\pm 1$ [1] where monopoles and dyons become massless.

The exact Kahler metric on the moduli space was computed in [21] and can be written in the following form:

[1]More precisely the two singularities are at the points $u=\pm\Lambda$.In this paper we are using units where the scale $\Lambda=1$.

$$ds^2 = g(u)dud\bar{u} = \mathrm{Im}\,(\tau\,(u)) \left|\frac{da\,(u)}{du}\right|^2 dud\bar{u} \tag{2.4}$$

where:

$$\begin{aligned}
\tau\,(u) &= \frac{\frac{da_D(u)}{du}}{\frac{da(u)}{du}} \\
a\,(u) &= \sqrt{2}\sqrt{u+1}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2};1;\frac{2}{u+1}\right) \\
a_D\,(u) &= i\frac{u-1}{2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\frac{1-u}{2}\right)
\end{aligned} \tag{2.5}$$

2.2 Soft breaking to $\mathcal{N} = 1$

We now consider adding a superpotential for the chiral multiplet, breaking $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. If the superpotential term is small, we can assume that we can still trust the effective IR description of the theory. In other words we assume that the Kahler metric on the moduli space is the same as in usual $\mathcal{N} = 2$ SW $U(1)$ theory and that the effect of the superpotential is to induce a superpotential $W(u)$ for the effective IR scalar field u. This superpotential will produce a potential on the moduli space equal to:

$$V\,(u) = g^{-1}\,(u)\,|W'(u)|^2 \tag{2.6}$$

where the Kahler metric is still given by the above relations (2.4).

The goal of this paper is to find a superpotential $W(u)$ which, once combined with the Kahler metric $g(u)$ given by the Seiberg-Witten solution, will induce a scalar potential (2.6) with a local minimum at some point of the moduli space. Of course this minimum must have nonzero energy if it has to correspond to a non-supersymmetric metastable vacuum. As noticed in [1], the simplest choice is the superpotential $W \sim Tr\Phi^2$, in terms of the UV fields, which takes the form $W(u) \sim u$ in terms of the fields in the IR effective theory:

$$\begin{aligned}
W &= \mu u \\
V\,(u) &= \mu^2 g^{-1}\,(u)
\end{aligned} \tag{2.7}$$

In this case the potential is equal to the inverse Kahler metric multiplied by a constant and we can see it plotted in figure 1.

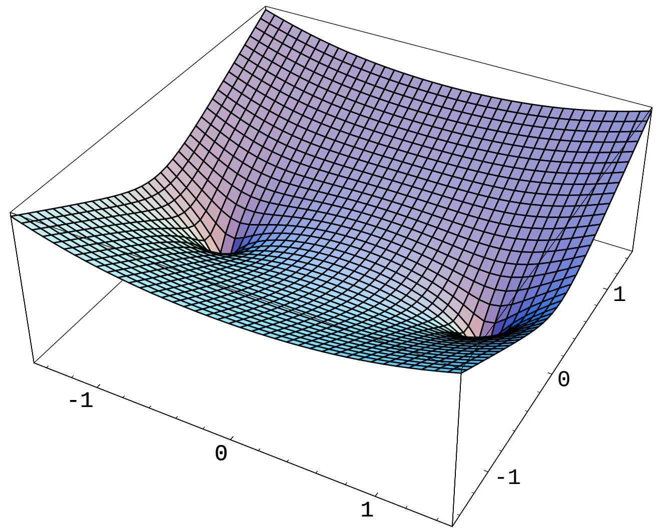

Figure 1: The potential due to the quadratic superpotential

As was pointed out in [1] in this case there are no metastable vacua in the moduli space. We can see the two usual supersymmetric vacua for the $\mathcal{N}=2$ theory broken to $\mathcal{N}=1$ by a mass term for the adjoint, which were also described in [21]. They correspond to the points on the moduli space where non perturbative objects became massless. We can also see a saddle point between them at the origin $u=0$.

The next step is to consider a more general superpotential. From the relation (2.6) we see that the scalar potential in the general case is the product of two factors: of the inverse Kahler metric and of the square of the derivative of the superpotential. As we saw above, the inverse Kahler metric has two global minima at $u=\pm 1$ where it is equal to zero corresponding to the supersymmetric vacua and a saddle point at $u=0$. We will try to find a superpotential that will produce a local minimum for the scalar potential at $u=0$, where the inverse Kahler metric has a saddle point.

Since the function $W(u)$ is holomorphic, it is easy to show that $|W'(u)|^2$

cannot have local minima except for the supersymmetric ones, when $W'(u) = 0$. However $|W'(u)|^2$ can have saddle points. By choosing $W(u)$ appropriately we can arrange that the saddle point of $|W'(u)|^2$ lies at $u = 0$, so that it coincides with the saddle point of the inverse Kahler metric. It is not difficult to show that the product of two functions which have a common saddle point at some u_0 will also have a stationary point at u_0. Moreover we can see that depending on the relative magnitudes of the second partial derivatives of the two functions, it is possible that the product can have a *local minimum* at u_0 even if the two factors only have saddle points at u_0.

In our case it turns out that the simplest possibility to consider is a superpotential of third order in u:

$$W = \mu \left(u + \lambda u^3\right) \tag{2.8}$$

We have set the quadratic term in u to zero, so that the saddle point of $|W'(u)|^2$ occurs exactly at the origin $u = 0$. To have a chance to get a metastable vacuum we need the stable and unstable directions of the saddle point of $|W'(u)|^2$ to be related to the stable and unstable directions of the saddle point of g^{-1} in such a way that the product of the two functions $V(u) = g^{-1}|W'(u)|^2$ has a stationary point with all directions stable. Otherwise we would again get a saddle point. This occurs if the coefficient of the third order term is positive.

In figure 2 we can see the plot of $|W'(u)|^2$, which indeed has a saddle point as we wanted. So now the potential is going to be the product of the two graphs above. The second derivatives of $|W'(u)|^2$ at the saddle point increase as we increase λ. So we expect if λ is too small, the saddle point to look like the first graph, while if λ is too large to look like the second graph, as we can understand by the following relation.

$$\begin{aligned}
\left.\frac{d^2 V}{d\,\mathrm{Re}\,(u)^2}\right|_0 &= \mu^2 \left(12\lambda g^{-1}\big|_0 + \left.\frac{d^2 g^{-1}}{d\,\mathrm{Re}\,(u)^2}\right|_0 \right) \\
\left.\frac{d^2 V}{d\,\mathrm{Im}\,(u)^2}\right|_0 &= \mu^2 \left(-12\lambda g^{-1}\big|_0 + \left.\frac{d^2 g^{-1}}{d\,\mathrm{Im}\,(u)^2}\right|_0 \right)
\end{aligned} \tag{2.9}$$

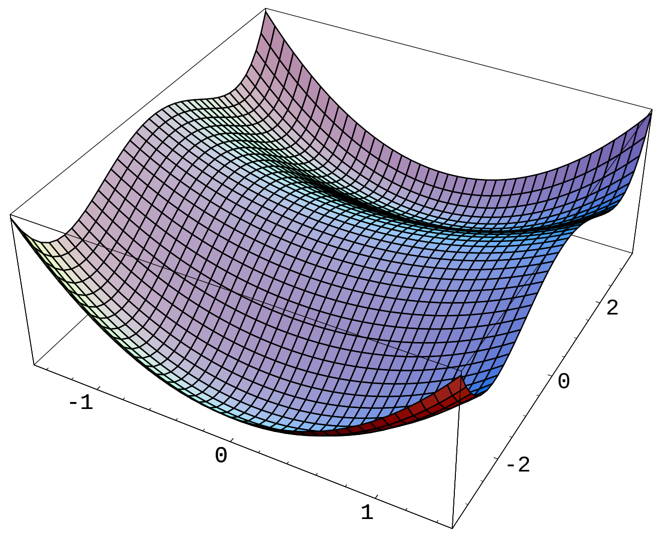

Figure 2: The potential by a third order superpotential with flat Kahler metric

Fortunately $\left|\frac{d^2g^{-1}}{d\,\mathrm{Re}(u)^2}\right| < \left|\frac{d^2g^{-1}}{d\,\mathrm{Im}(u)^2}\right|$ at the origin, so actually there is a range of λ for which the origin becomes a local minimum. Using properties of the hypergeometric functions we find:

$$\lambda_- < \lambda < \lambda_+ \tag{2.10}$$

where

$$\lambda_\pm = \frac{1}{24}\left[1 \pm \left(\frac{\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{5}{4}\right)}\right)^4\right] \tag{2.11}$$

For example, for superpotential equal to:

$$W = 0.01\left(u + \frac{1}{24}u^3\right) \tag{2.12}$$

we get the picture of figure 3. If one zooms at the x-saddle point (figures 4 and 5) sees the meta-stable vacuum.

Let's make a few comments on this potential. There are four supersymmetric vacua. Two are the Seiberg Witten ones at $u = \pm 1$. The other two are the ones

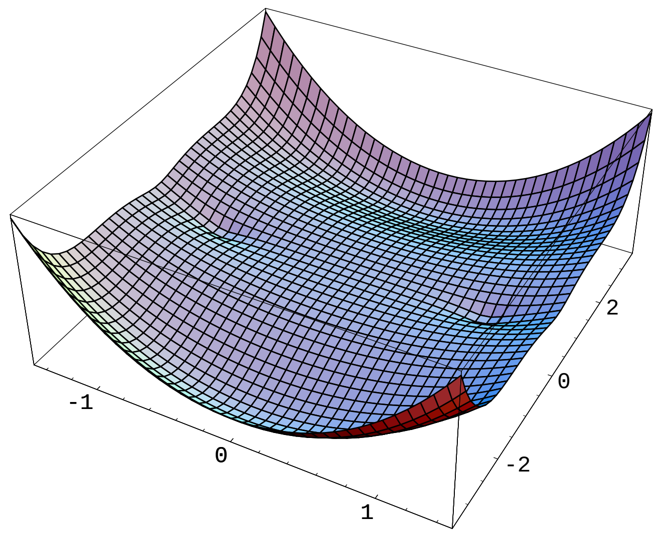

Figure 3: The full potential, a metastable vacuum exists at the origin

induced by the superpotential, they are the zero's of $\left|\frac{\partial W}{\partial u}\right|^2$. Their position is $u = \pm i\frac{1}{\sqrt{3\lambda}}$. The metastable vacuum lies at $u = 0$, and has four possible decays towards the four supersymmetric vacua.

Another thing is that we can make the $\mathcal{N} = 2$ breaking superpotential as small we like by making μ small. Changing μ results just in multiplication of the potential with a constant thus not changing the picture we saw above. So the assumption we made that there is no significant change induced to the Kahler metric by this superpotential can be satisfied.

The picture of the potential, as we move in this parameter region changes as follows.

For λ close to λ_- as seen in figure 6 the metastable vacuum minimum is elongated along the imaginary axis, looking more possible to decay towards the Seiberg-Witten vacua.

For λ close to λ_+ as seen in figure 7 the metastable vacuum minimum is elongated along the real axis, looking more possible to decay towards the superpotential vacua.

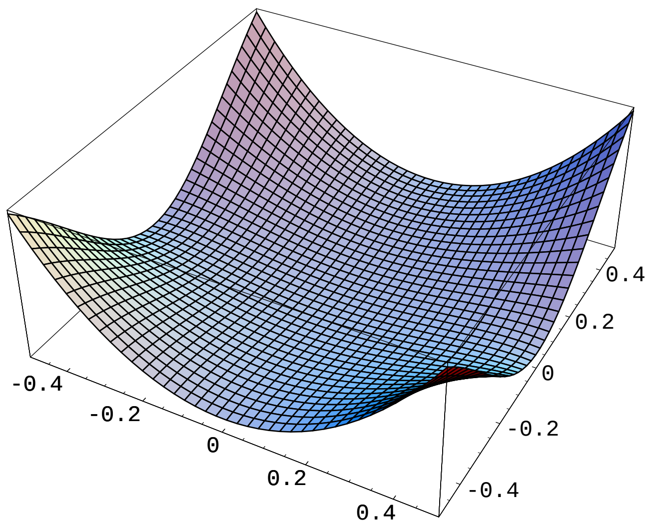

Figure 4: Close-up on the x-saddle point

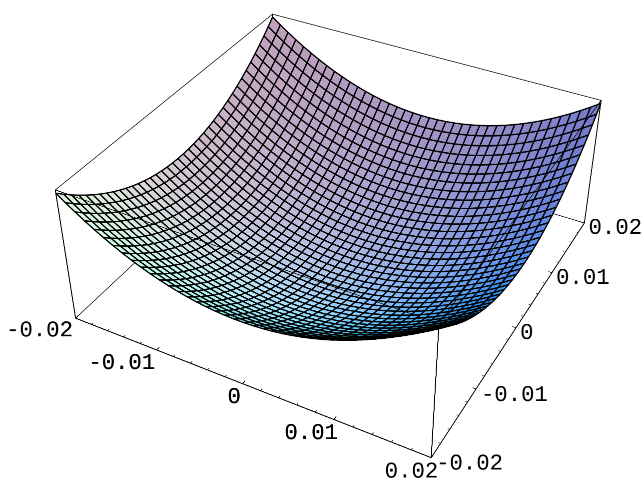

Figure 5: A closer close-up on the x-saddle point

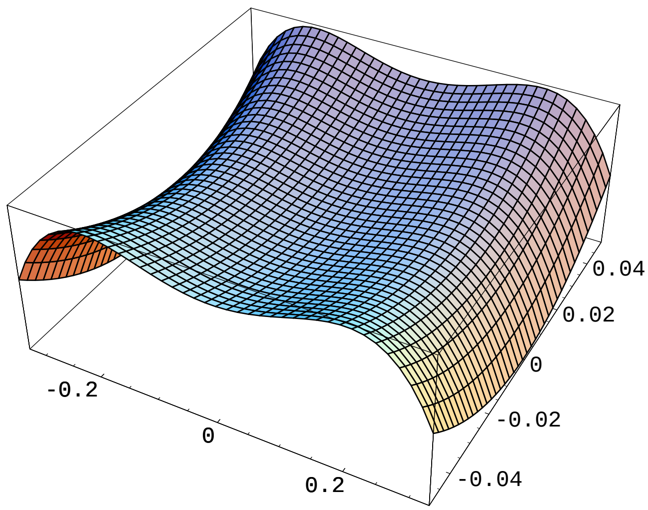

Figure 6: The area of the metastable vacuum for λ close to λ_-

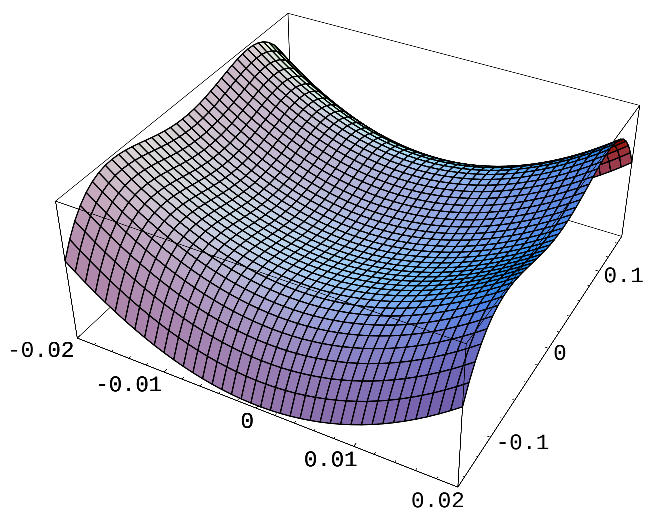

Figure 7: The area of the metastable vacuum for λ close to λ_+

2.3 Lifetime of the metastable vacuum

In order to estimate the lifetime of the metastable vacuum, we use the triangular approximation [27]. The thin wall approximation [28] is not a good approximation in our case, as the ratio of the barrier width and barrier height is not small enough.

Let's say the position and potential at the true vacuum, the false vacuum and the peak of the potential between them are ϕ_-, ϕ_+, ϕ_T and V_-, V_+, V_T respectively. Then we define

$$\begin{aligned}
\Delta\phi_\pm &= |\phi_T - \phi_\pm| \\
\Delta V_\pm &= V_T - V_\pm \\
\lambda_\pm &= \frac{\Delta V_\pm}{\Delta\phi_\pm} \\
c &= \frac{\lambda_-}{\lambda_+}
\end{aligned} \tag{2.13}$$

and the decay rate is given by:

$$\frac{\Gamma}{V} \sim A e^{-B} \tag{2.14}$$

B is given by,

$$B = \frac{32\pi^2}{3} \frac{1+c}{\left(\sqrt{1+c}-1\right)^4} \left(\frac{\Delta\phi_+^4}{\Delta V_+}\right) \tag{2.15}$$

Applying the above in our case for both possible decays we get:

$$\begin{aligned}
c_{SW} &= \frac{V_{T,SW}}{V_{T,SW} - V_0} \frac{\phi_{T,SW}}{1 - \phi_{T,SW}} \\
c_W &= \frac{V_{T,W}}{V_{T,W} - V_0} \frac{\phi_{T,W}}{\frac{1}{\sqrt{3\lambda}} - \phi_{T,W}}
\end{aligned} \tag{2.16}$$

The numerical results for the B factor as function of the parameter λ for $\mu = 1$ in this approximation are shown in figures 8 and 9.

We observe that generally decay towards the SW vacua is favorable. This is because the superpotential vacua are more distant. We also observe that for $\lambda = \lambda_-$ the field flows directly towards the SW vacuum, and for $\lambda = \lambda_+$ towards

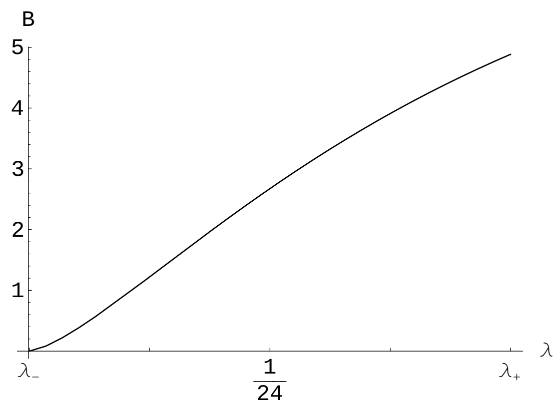

Figure 8: B factor for the decay towards the SW vacuum

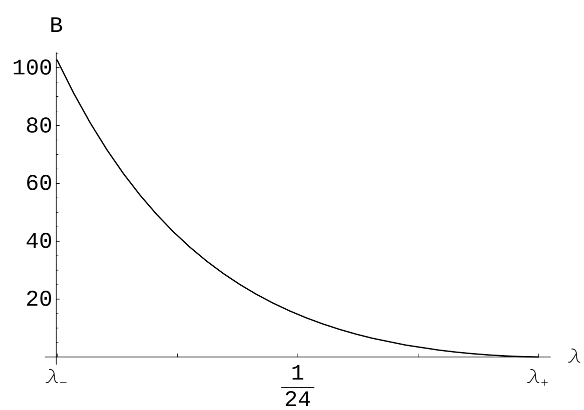

Figure 9: B factor for the decay towards the superpotential vacuum

the superpotential vacuum as expected. There is actually a value for λ around 0.0457 where the two decays are equally favorable.

Above we found the decay rate in the triangular approximation for $\mu = 1$. If one assumes an arbitrary μ what happens is:

$$\begin{aligned} \phi_\pm &\to \phi_\pm \\ V_\pm &\to \mu^2 V_\pm \\ c &\to c \\ B &\to \mu^{-2} B \end{aligned} \tag{2.17}$$

meaning that we can make the metastable vacuum as long lived as we want by making μ as small as necessary. Notice that our analysis is reliable exactly for small μ as noticed above.

2.4 Spectrum at the metastable vacuum

Obviously as there is a $U(1)$ gauge symmetry remaining everywhere in the moduli space, the gauge boson remains massless. The gaugino is also massless.

The mass of the fermion partner of u is given by the second derivative of the superpotential. However as we have not added any second order terms in the superpotential the second derivative at the origin is zero. That means that the fermion partner of u remains also massless.

Last we can calculate the scalar masses directly from the second derivatives of the scalar potential. We are careful to divide with the inverse Kahler metric as our fields are not canonical. From equations 2.9 we find:

$$\begin{aligned} m^2_{\phi\,\mathrm{Re}} &= 12\mu^2(\lambda - \lambda_-) \\ m^2_{\phi\,\mathrm{Im}} &= 12\mu^2(\lambda_+ - \lambda) \end{aligned} \tag{2.18}$$

The supertrace equals

$$\sum m_B^2 - \sum m_F^2 = 12\mu^2\left(\lambda_+ - \lambda_-\right) = \left(\frac{\Gamma\left(\frac{3}{4}\right)}{2\Gamma\left(\frac{5}{4}\right)}\right)^4 \mu^2 \tag{2.19}$$

and does not depend on λ as expected (because the third derivatives of the

superpotential give off-diagonal contributions to the boson mass matrix). The supertrace is non zero as the Kahler metric has non vanishing curvature.

As we have two massless fermions it is a fair question which is the goldstino. The gaugino does not interfere to the process of supersymmetry breaking and this is actually the reason it remains massless, or better its mass does not split with the mass of the gauge boson. The actual goldstino is the fermion partner of the scalar u

2.5 Origin of the non-renormalizable superpotential

We know that the non-trivial Kahler metric we use occurs in the low energy limit of a theory well defined in the UV, specifically the $\mathcal{N} = 2$ $SU(2)$ SYM without flavors. We would like to know if it is also possible to get also the non-renormalizable superpotential as low energy effective superpotential of a UV complete theory. We have to be careful though, so we don't alter the $\mathcal{N} = 2$ theory in such a way that the SW Kahler metric calculation is not reliable anymore.

It is actually simple to get the appropriate term by adding in the $\mathcal{N} = 2$ SYM a gauge singlet massive hypermultiplet ($\tilde{M}$, M). In order to get something non trivial for the adjoint we need to couple it with the heavy hypermultiplet. The only possible gauge invariant and renormalizable term is $MTr\Phi^2$. We also add a third order superpotential for the other chiral multiplet, so:

$$W = aMTr\Phi^2 + b\tilde{M}^3 + mM\tilde{M} \tag{2.20}$$

For the reliability of the Kahler metric calculation we need the mass term has to be large so, the extra hypermultiplet can be integrated out at high enough energies. The other two parameters have to be small, so they don't alter the structure of the $\mathcal{N} = 2$ at low energies.

In order to integrate out the massive hypermultiplet, we need to find the equations of motion of the massive fields, as they occur from the superpotential, and substitute the massive hypermultiplet in the superpotential, using these

equations. We find:

$$W_{eff} = -\frac{a^3 b}{m^3}\left(Tr\Phi^2\right)^3 \tag{2.21}$$

Thus at low energies we get the required superpotential. As the coefficient of this term in our analysis was $\mu\lambda$, and we could make μ as small as we like, that means that we can make a, b and m^{-1} as small as needed for the validity of the calculation.

3 Higher rank groups

3.1 $SU(3)$ $\mathcal{N}=2$ SYM with flavor

In the case of $SU(3)$ the moduli space is a two dimensional complex manifold parametrized by the two complex numbers u, v. The metric on the moduli space can be computed from the curve C [23, 24]:

$$y^2 = (x^3 - ux - v)^2 - 1 \tag{3.1}$$

where we have set the strong coupling scale $\Lambda = 1$.

When we add flavor the curve takes the form [25, 26]:

$$y^2 = (x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x + m_i) \tag{3.2}$$

where m_i are the masses of the hypermultiplets.

Using Riemann bilinear identities we can write the Kahler metric elements as:

$$\begin{aligned} g_{u,\bar{u}} &= \int_C \omega_u \wedge \bar{\omega_u}, \quad g_{v,\bar{v}} = \int_C \omega_v \wedge \bar{\omega_v} \\ g_{u,\bar{v}} &= \int_C \omega_u \wedge \bar{\omega_v}, \quad g_{v,\bar{u}} = \int_C \omega_v \wedge \bar{\omega_u} \end{aligned} \tag{3.3}$$

where

$$\omega_v = \frac{1}{y}dx, \qquad \omega_u = \frac{x}{y}dx \tag{3.4}$$

Then the potential equals

$$V = g^{u,\bar{u}}|W_u|^2 + g^{v,\bar{v}}|W_v|^2 + 2\,\mathrm{Re}(g^{u,\bar{v}}W_u W_v^*) \tag{3.5}$$

where

$$W_u = \frac{\partial W}{\partial u}, \quad W_v = \frac{\partial W}{\partial v} \tag{3.6}$$

This is a function of the variables u, v, the parameters m_i, and the derivatives of the superpotential W_u, W_v. For any given superpotential and values of the parameters we have to find local minima in terms of the variables u, v.

Unfortunately as we can see from (3.5) we have to invert the Kahler metric to write down the potential and this complicates things a little bit. Inverting the Kahler metric, we can write the potential in the form:

$$V = \frac{1}{g_{u\bar{u}}g_{v\bar{v}} - |g_{u\bar{v}}|^2} \times (g_{v\bar{v}}|W_u|^2 + g_{u\bar{u}}|W_v|^2 + 2\,\mathrm{Re}(g_{u\bar{v}}W_u\bar{W}_v)) \tag{3.7}$$

The potential has the form:

$$V = \frac{h_1}{h_2} \tag{3.8}$$

where

$$\begin{aligned} h_1 &= (g_{v\bar{v}}|W_u|^2 + g_{u\bar{u}}|W_v|^2 + 2\,\mathrm{Re}(g_{u\bar{v}}W_u\bar{W}_v)) \\ h_2 &= g_{u\bar{u}}g_{v\bar{v}} - |g_{u\bar{v}}|^2 \end{aligned} \tag{3.9}$$

The coefficients of the Kahler metric that we want to compute to determine the effective potential have the form:

$$g_{a\bar{b}}(z) = \int dx d\bar{x} \frac{g(x)}{|P(x)|} \tag{3.10}$$

where $g(x)$ is either $|x|^2$ or x or constant and $P(x)$ is the right hand side of 3.2

We are interested in calculating the above in locations of the moduli space where $P(x)$ has a double root. We show in appendix A although the metric

elements go to zero as the above integral diverges, the potential is actually finite and has the form:

$$V_{eff} = \frac{h_1}{h_2} = \frac{|W_u|^2 + |W_v|^2|r|^2 + 2\,\mathrm{Re}(rW_uW_v^*)}{\int dx d\bar{x} \frac{1}{|P'(x)|}} \tag{3.11}$$

where r is the double root and

$$P(x) = P'(x)(x-r)^2 \tag{3.12}$$

which is simplified, but more importantly, it is finite.

So to minimize the potential on the singular submanifold we have to maximize:

$$\int dx d\bar{x} \frac{1}{|P'(x)|} \tag{3.13}$$

where remember that $P'(x)$ is the factorized curve. Except for this expression being simpler to calculate numerically than the initial one, it is possible, as the factored polynomial is a lower rank one, that the potential on the singular submanifold matches the potential of a lower rank theory. We will show later that this actually happens.

3.2 $SU(3)$ with 2 massless flavors

In $SU(3)$ with even number of flavors or with no flavors, the sixth order polynomial factorizes to two third order polynomials making analysis much simpler. We do this analytically for general even number of massive flavors in appendix B. Here we use the results only for two massless flavors.

In $SU(3)$ with 2 massless flavors we have:

$$P = \left(x^3 - ux - v\right)^2 - x^2 \tag{3.14}$$

We observe that at $v = 0$ the polynomial has a double root equal to zero. According to the previous analysis the potential on this surface equals:

$$V_{eff} = \frac{|W_u|^2}{\int dx d\bar{x} \frac{1}{|P'(x)|}} \tag{3.15}$$

where

$$P' = \left(x^2 - u\right)^2 - 1 \tag{3.16}$$

As the double root in this case is $r = 0$ the potential on the submanifold, as seen from equation 3.11, the potential depends only on W_u.

So the potential is identical with the $SU(2)$ theory without flavors, which we analyzed in previous section. So it suffices to show that at $u = 0$ the potential increases as we move away from the singular submanifold, to show that we can again construct a metastable vacuum, using the exact same softly breaking superpotential.

We saw that W_v does not make any difference in the potential on the submanifold. However it is interesting to see what effects such a term has close to the submanifold. For simplicity let's assume a superpotential term:

$$W_v = \kappa v \tag{3.17}$$

We show in appendix C that the potential induced by this term for small v is approximately:

$$V_v \sim \frac{(1 - |2v|)}{\log(|2v|)} \tag{3.18}$$

This potential strongly constrains the field in the singular submanifold, as one can see in figure 10. So once we turn on the W_v term, the $v = 0$ plane becomes locally stable.

Moreover this result combined with the result of the previous section, means that $\mathcal{N} = 2$ $SU(3)$ SYM with two massless flavors, and softly breaking to $\mathcal{N} = 1$ superpotential

$$W = \mu\left(u + \lambda u^3\right) + \kappa v \tag{3.19}$$

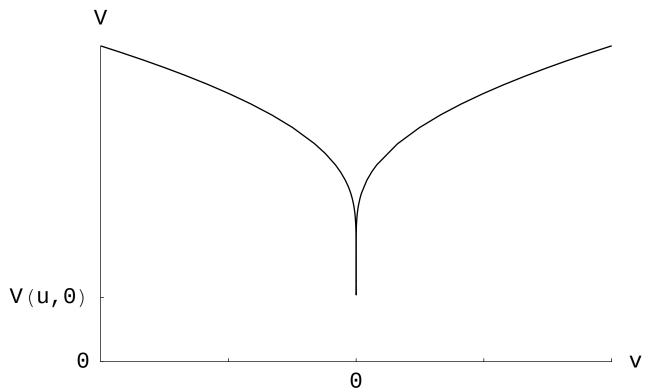

Figure 10: Potential in a perpendicular to the singular submanifold direction

has a metastable vacuum at $u = v = 0$, for the same range of λ as in the pure $SU(2)$ case.

3.3 Comments on higher rank groups

It seems clear, that the results analyzed in previous two sections and the appendices can be generalized to higher rank groups. $SU(N)$ theory with N_f massless flavors can be reduced to $SU(N-1)$ with $N_f - 2$ massless flavors on the subspace where the highest order moduli is zero. Moreover this subspace can be locally stabilized by turning on a superpotential term linear in this moduli.

This means that we can construct a metastable vacuum in the origin of the moduli space of all $SU(N)$ theories with $2N-4$ massless flavors.

Constructions of metastable vacua in theories with large flavor symmetries can have interesting phenomenological applications in particular in building models of direct gauge mediation. (See for example [29] for references)

We can also show that $SU(3)$ with two massive flavor reduces to $SU(2)$ on the $v = 0$ submanifold with the only additional change of a shift of the moduli u by $\frac{3}{4}m^2$ (See appendix B. This means that we are again able to build the same metastable vacuum if we simply shift u in the appropriate superpotential too. However a more careful analysis for the local stability of the submanifold

is needed.

Higher rank groups without flavors are studied in [20].

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A Potential on singular submanifolds

A.1 The potential in $SU(3)$ $\mathcal{N}=2$ SYM with flavor

We saw in section 3 that the potential has the form:

$$V = \frac{h_1}{h_2} \tag{A.1}$$

where

$$\begin{gathered} h_1 = (g_{v\bar{v}}|W_u|^2 + g_{u\bar{u}}|W_v|^2 + 2\,\mathrm{Re}(g_{u\bar{v}}W_u\bar{W}_v)) \\ h_2 = g_{u\bar{u}}g_{v\bar{v}} - |g_{u\bar{v}}|^2 \end{gathered} \tag{A.2}$$

The function h_1 depends on u, v, m_i and also W_u, W_v, while h_2 is independent of W_u, W_v.

The function h_1 can be written in the form:

$$h_1 = \int dx d\bar{x} \frac{|W_u|^2 + |x|^2|W_v|^2 + 2\,\mathrm{Re}(xW_uW_v^*)}{y\bar{y}} \tag{A.3}$$

or

$$h_1 = \int dx d\bar{x} \frac{|W_u + xW_v|^2}{|(x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x+m_i)|} \tag{A.4}$$

To sum up the moduli space is parametrized by two complex variables u, v. The effective scalar potential on the moduli space is equal to:

$$V_{eff} = \frac{h_1(u,v)}{h_2(u,v)} \tag{A.5}$$

where the functions $h_1(u,v)$, $h_2(u,v)$ also depend on W_u, W_v and the masses of the quarks and can be written in terms of integrals over the x-plane as:

$$h_1 = \int dx d\bar{x} \frac{|W_u + xW_v|^2}{|(x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x + m_i)|} \tag{A.6}$$

$$\begin{aligned} h_2 = &\left(\int dx d\bar{x} \frac{|x|^2}{|(x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x + m_i)|} \right) \times \\ &\times \left(\int dx d\bar{x} \frac{1}{|(x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x + m_i)|} \right) \\ &- \left| \int dx d\bar{x} \frac{x}{|(x^3 - ux - v)^2 - \prod_{i=1}^{N_f}(x + m_i)|} \right|^2 \end{aligned} \tag{A.7}$$

A.2 Toy Model

We are interested in calculating the potential at submanifolds of the moduli space where the polynomial has a double root. Obviously the above integrals diverge on such submanifolds. However we will show that the inverse Kahler metric is finite. We can use the following way of regularizing the integral:

$$k(z) = \int \frac{f(x)}{|x||x - z|} \tag{A.8}$$

with the function $f(x)$ smooth near zero and falling off fast enough at infinity. Again the integral diverges logarithmically as $z \to 0$. We add and subtract the function $\frac{f(0)}{|x|(|x|+1)(|x|+|z|)}$ and we have:

$$\begin{aligned} k(z) = &\int \left(\frac{f(x)}{|x||x - z|} - \frac{f(0)}{|x|(|x| + 1)(|x| + |z|)} \right) \\ &+ \int \frac{f(0)}{|x|(|x| + 1)(|x| + |z|)} \end{aligned} \tag{A.9}$$

this is of the form:

$$k(z) = h(z) + g(z) \tag{A.10}$$

where we can compute the second function exactly:

$$g(z) = 2\pi f(0)\frac{\log(z)}{1-z} \tag{A.11}$$

and the first function is finite for all values of z. So the only divergence as $z \to 0$ comes from $g(z)$ and we can write:

$$k(z) = -2\pi f(0)\log(z) + \int \left(\frac{f(x)}{|x|^2} - \frac{f(0)}{|x|^2(|x|+1)} \right) + \mathcal{O}(z) \tag{A.12}$$

Of course if the double root is not at zero but at an other point $x = r$ we have the obvious generalization:

$$\begin{aligned} &\int \frac{f(x)}{|x-r||x-r-z|} \\ &\quad = -2\pi f(r)\log(z) + \int \left(\frac{f(x)}{|x-r|^2} - \frac{f(r)}{|x-r|^2(|x-r|+1)} \right) + \mathcal{O}(z) \end{aligned} \tag{A.13}$$

A.3 Application to our system

The SW curve for our system (SU(3) with flavor) has the form:

$$y^2 = P(X) \tag{A.14}$$

where $P(x) = \prod_i (x - r_i)$ is some polynomial in x, whose coefficients depend on the moduli space coordinates u, v.

Let's consider a point on the moduli space (u_0, v_0) where two and only two roots coincide, let's say to the value r.

This means that at that point the polynomial P factorizes to:

$$P(x) = P'(x)(x-r)^2 \tag{A.15}$$

where P' is a polynomial of degree two less than P and with all roots distinct. Now let's move away from the singular point (u_0, v_0) in such a way that the

distance between the ex-coincident roots is z, where by z we denote the absolute value of the distance.

The coefficients of the Kahler metric that we want to compute to determine the effective potential have the form:

$$g_{a\bar{b}}(z) \sim \int dx d\bar{x} \frac{g(x)}{|P(x)|} = \int dx d\bar{x} \frac{f(x)}{|(x-r)(x-r-z)|} \tag{A.16}$$

where $g(x)$ is either $|x|^2$ or x or constant and $f(x)$ contains both $g(x)$ and the polynomial $P'(x)$.

As we can see this integral diverges logarithmically as $z \to 0$.

We want to write the diverging integrals in the form:

$$g_{a\bar{b}}(z) = -a \log(z) + b + \mathcal{O}(z) \tag{A.17}$$

and determine the constants a and b in terms of $f(x)$:

Using our previous trick we write:

$$\begin{aligned} g_{a\bar{b}}(z) = &-2\pi f(r) \log(z) \\ &+ \int \left(\frac{f(x)}{|x-r|^2} - \frac{f(r)}{|x-r|^2(|x-r|+1)} \right) + \mathcal{O}(z) \end{aligned} \tag{A.18}$$

Let's do it for all the coefficients of the Kahler metric:

$$g_{u\bar{u}} = \int \frac{|x|^2}{|P(x)|} = \int \frac{|x|^2}{|P'_z(x)||x-r||x-r-z|} \tag{A.19}$$

where the subscript z in P' means the P' maybe depends slowly on z but in the limit $z \to 0$ this dependence is irrelevant.

As before we have:

$$\begin{aligned} g_{u\bar{u}} = &-2\pi \frac{|r|^2}{|P'(r)|} \log(z) \\ &+ \int \left(\frac{|x|^2}{|x-r|^2 |P'(x)|} - \frac{|r|^2}{|P'(r)||x-r|^2(|x-r|+1)} \right) + \mathcal{O}(z) \end{aligned} \tag{A.20}$$

Similarly we find:

$$\begin{aligned} g_{v\bar{v}} &= -2\pi \frac{1}{|P'(r)|} \log(z) \\ &+ \int \left(\frac{1}{|x-r|^2 |P'(x)|} - \frac{1}{|P'(r)||x-r|^2(|x-r|+1)} \right) + \mathcal{O}(z) \end{aligned} \tag{A.21}$$

and finally:

$$\begin{aligned} g_{u\bar{v}} &= -2\pi \frac{r}{|P'(r)|} \log(z) \\ &+ \int \left(\frac{x}{|x-r|^2 |P'(x)|} - \frac{r}{|P'(r)||x-r|^2(|x-r|+1)} \right) + \mathcal{O}(z) \end{aligned} \tag{A.22}$$

In the denominator of the effective potential we have the combination $h_2 = g_{u\bar{u}} g_{v\bar{v}} - |g_{u\bar{v}}|^2$. The terms that go like $(\log z)^2$ cancel, and we collect all terms proportional to $\log(z)$ to find:

$$\begin{aligned} h_2 = -2\pi \frac{1}{|P'(r)|} \log(z) \times& \\ \times \Bigg[\Bigg(\int & \frac{|x|^2}{|x-r|^2 |P'(x)|} - \frac{|r|^2}{|P'(r)||x-r|^2(|x-r|+1)} \Bigg) \\ + |r|^2 \int & \left(\frac{1}{|x-r|^2 |P'(x)|} - \frac{1}{|P'(r)||x-r|^2(|x-r|+1)} \right) \\ -2 \,\mathrm{Re} \Bigg(r^* \int & \left(\frac{x}{|x-r|^2 |P'(x)|} - \frac{r}{|P'(r)||x-r|^2(|x-r|+1)} \right) \Bigg) \Bigg] \end{aligned} \tag{A.23}$$

Now all three integrals are finite, so we can combine them and simplify the integrand and we find:

$$h_2 = -2\pi \frac{1}{|P'(r)|} \log(z) \int dx d\bar{x} \frac{1}{|P'(x)|} + \mathcal{O}(1) \tag{A.24}$$

The numerator is $h_1 = |W_u|^2 g_{v\bar{v}} + |W_v|^2 g_{u\bar{u}} + 2\,\mathrm{Re}\,|W_u W_v^*|^2 g_{v\bar{u}}$ We keep only the log diverging part so we find:

$$h_1 = -2\pi \frac{1}{|P'(r)|} \log(z) \left(|W_u|^2 + |W_v|^2 |r|^2 + 2\,\mathrm{Re}(r W_u W_v^*) \right) + \mathcal{O}(1) \tag{A.25}$$

Combining our results we find that the effective potential on the submanifold where two roots coincide has the simple form:

$$V_{eff} = \frac{h_1}{h_2} = \frac{|W_u|^2 + |W_v|^2|r|^2 + 2\,\mathrm{Re}(rW_uW_v^*)}{\int dx d\bar{x} \frac{1}{|P'(x)|}} \tag{A.26}$$

which is simplified, but more importantly, it is finite.

B $SU(3)$ with $N_f = 2k$

In $SU(3)$ with even number of flavors or with no flavors, the sixth order polynomial factorizes to two third order polynomials making analysis much simpler. For this factorization to be possible it suffices that hypermultiplets come in pairs of equal mass. However in the following analysis we will assume that they are all equal for simplicity.

In $SU(3)$ with $2k$ flavors (including $k = 0$) we have:

$$P_{2k} = \left(x^3 - ux - v\right)^2 - (x+m)^{2k} \tag{B.1}$$

So we can write:

$$P_{2k} = P_{2k+}P_{2k-} \tag{B.2}$$

where

$$P_{2k\pm} = \left(x^3 - ux - v\right) \pm (x+m)^k \tag{B.3}$$

One possibility is that two roots of P_- or two roots of P_+ coincide. However here we will concentrate in a much simpler to analyze case the case where one root of P_- coincides with one root of P_+. We are mainly interested in this case as we will be able to show in next section, that the submanifold of the moduli space where this happens is energetically preferred at least locally.

Obviously one root of P_- cannot coincide with one root of P_+ in the pure $SU(3)$ case. In the other two cases the only possibility is that the common root is $x = -m$. This means that the singular submanifold is:

$$v = um - m^3 \tag{B.4}$$

On the submanifold the polynomials can be written as:

$$P_\pm = (x+m)\left[\left(x^2 - mx + m^2 - u\right) \pm 1\right] \tag{B.5}$$

for the two flavors case, and:

$$P_\pm = (x+m)\left[\left(x^2 - mx + m^2 - u\right) \pm (x+m)\right] \tag{B.6}$$

for the four flavor case.

This means that:

$$P' = \left(x^2 - mx + m^2 - u\right)^2 - 1 \tag{B.7}$$

for two flavors and

$$P' = \left(x^2 - mx + m^2 - u\right)^2 - (x+m)^2 \tag{B.8}$$

for four flavors.

As x is about to be integrated on the whole complex plane, and the only x-dependence of the integrand is in the polynomial, we can always shift it by a constant. So we can eliminate the linear terms in the first parenthesis shifting x by $\frac{m}{2}$. We get:

$$P' = \left(x^2 + \frac{3}{4}m^2 - u\right)^2 - 1 \tag{B.9}$$

and

$$P' = \left(x^2 + \frac{3}{4}m^2 - u\right)^2 - \left(x + \frac{3}{2}m\right)^2 \tag{B.10}$$

The last forms are well known. The first is the polynomial for the pure $SU(2)$ where we have shifted u by $\frac{3}{4}m^2$ and the second is the $SU(2)$ with u shifted by the same amount and two flavors of mass $\frac{3}{2}m$.

Specifically for $m = 0$ the first case simplifies to the most known

$$P' = \left(x^2 - u\right)^2 - 1 \tag{B.11}$$

and the singular submanifold in this case is simply $v = 0$. As the double root in this case is $x = 0$ the potential on the submanifold, as seen from equation 3.11, the potential depends only on the part of the superpotential that depends only on u.

$$V_{eff} = \frac{|W_u|^2}{\int dx d\bar{x} \frac{1}{|P'(x)|}} \tag{B.12}$$

C Stability of the singular submanifold

In previous appendix we showed that we can have an easier expression of the potential in a submanifold of the $v = 0$ moduli space of $SU(3)$ gauge theory with $2k$ flavors. Now let's see if the system has a preference to lay in this singular submanifold. We saw in previous section that the v-part of the superpotential does not make any difference in the potential on the submanifold. However it is interesting to see what effects such a term has close to the submanifold. For simplicity let's assume a superpotential term:

$$W_v = \kappa v \tag{C.1}$$

When one moves a little away from the $v = 0$ plane, the roots of the two polynomials slightly move away from zero. For small v the roots are also small, and one can neglect the cubic term giving us:

$$r_\pm \simeq \frac{v}{u \mp 1} \Rightarrow z = |r_+ - r_-| \simeq 2\,|v| \tag{C.2}$$

So the difference between the two roots depends linearly on v. This approximation does not hold only close to $u = \pm 1$, but these are the SUSY SW vacua, so they are stable anyway.

We saw in previous sections that in general wherever two roots of the polynomial coincide the Kahler metric vanishes logarithmically. We also saw that the inverse Kahler metric element does not have to vanish as the divergent parts cancel at the inversion process.

We remind that the logarithmically divergent parts of Kahler metric elements were:

$$\begin{aligned} g_{u\bar{u}} &= -2\pi\frac{|r|}{|P'(r)|}\frac{\log(z)}{1-z}+\mathcal{O}(1) \\ g_{v\bar{v}} &= -2\pi\frac{1}{|P'(r)|}\frac{\log(z)}{1-z}+\mathcal{O}(1) \\ g_{u\bar{v}} &= -2\pi\frac{r}{|P'(r)|}\frac{\log(z)}{1-z}+\mathcal{O}(1) \end{aligned} \tag{C.3}$$

where z is the magnitude of the difference of the two approaching roots, and r is the double root. However in our case the double root is equal to zero, meaning that there are no divergences in $g_{u\bar{u}}$ and $g_{u\bar{v}}$. That means that if we go a little bit away of the $v=0$ submanifold, the contribution of the new superpotential term will be:

$$V_v = \frac{g_{u\bar{u}}\,|\kappa|^2}{g_{u\bar{u}}g_{v\bar{v}}-|g_{u\bar{v}}|} \sim \frac{(1-|2v|)}{\log(|2v|)} \tag{C.4}$$

plus something similar coming from the $g_{u\bar{v}}$ term

References

[1] K. Intriligator, N. Seiberg and D. Shih, JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].

[2] R. Kitano, Phys. Rev. D **74**, 115002 (2006) [arXiv:hep-ph/0606129].

[3] T. Banks, arXiv:hep-ph/0606313.

[4] M. Schmaltz and R. Sundrum, JHEP **0611**, 011 (2006) [arXiv:hep-th/0608051].

[5] M. Dine and J. Mason, arXiv:hep-ph/0611312.

[6] R. Kitano, H. Ooguri and Y. Ookouchi, Phys. Rev. D **75**, 045022 (2007) [arXiv:hep-ph/0612139].

[7] H. Murayama and Y. Nomura, Phys. Rev. Lett. **98**, 151803 (2007) [arXiv:hep-ph/0612186].

[8] C. Csaki, Y. Shirman and J. Terning, arXiv:hep-ph/0612241.

[9] K. Intriligator, N. Seiberg and D. Shih, arXiv:hep-th/0703281.

[10] S. Franco and A. M. .. Uranga, JHEP **0606**, 031 (2006) [arXiv:hep-th/0604136].

[11] H. Ooguri and Y. Ookouchi, Nucl. Phys. B **755**, 239 (2006) [arXiv:hep-th/0606061].

[12] H. Ooguri and Y. Ookouchi, Phys. Lett. B **641**, 323 (2006) [arXiv:hep-th/0607183].

[13] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].

[14] I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, JHEP **0611**, 088 (2006) [arXiv:hep-th/0608157].

[15] R. Argurio, M. Bertolini, S. Franco and S. Kachru, JHEP **0701**, 083 (2007) [arXiv:hep-th/0610212].

[16] M. Aganagic, C. Beem, J. Seo and C. Vafa, arXiv:hep-th/0610249.

[17] A. Giveon and D. Kutasov, arXiv:hep-th/0703135.

[18] R. Argurio, M. Bertolini, S. Franco and S. Kachru, arXiv:hep-th/0703236.

[19] T. Kawano, H. Ooguri and Y. Ookouchi, arXiv:0704.1085 [hep-th].

[20] H. Ooguri, Y. Ookouchi and C. S. Park, arXiv:0704.3613 [hep-th].

[21] N. Seiberg and E. Witten, Nucl. Phys. B **426**, 19 (1994) [Erratum-ibid. B **430**, 485 (1994)] [arXiv:hep-th/9407087].

[22] N. Seiberg and E. Witten, Nucl. Phys. B **431**, 484 (1994) [arXiv:hep-th/9408099].

[23] A. Klemm, W. Lerche, S. Yankielowicz and S. Theisen, Phys. Lett. B **344**, 169 (1995) [arXiv:hep-th/9411048].

[24] P. C. Argyres and A. E. Faraggi, Phys. Rev. Lett. **74**, 3931 (1995) [arXiv:hep-th/9411057].

[25] A. Hanany and Y. Oz, Nucl. Phys. B **452**, 283 (1995) [arXiv:hep-th/9505075].

[26] I. M. Krichever and D. H. Phong, J. Diff. Geom. **45**, 349 (1997) [arXiv:hep-th/9604199].

[27] M. J. Duncan and L. G. Jensen, Phys. Lett. B **291**, 109 (1992).

[28] S. R. Coleman, Phys. Rev. D **15**, 2929 (1977) [Erratum-ibid. D **16**, 1248 (1977)].

[29] G. F. Giudice and R. Rattazzi, Phys. Rept. **322**, 419 (1999) [arXiv:hep-ph/9801271].